

Subdivision Surfaces

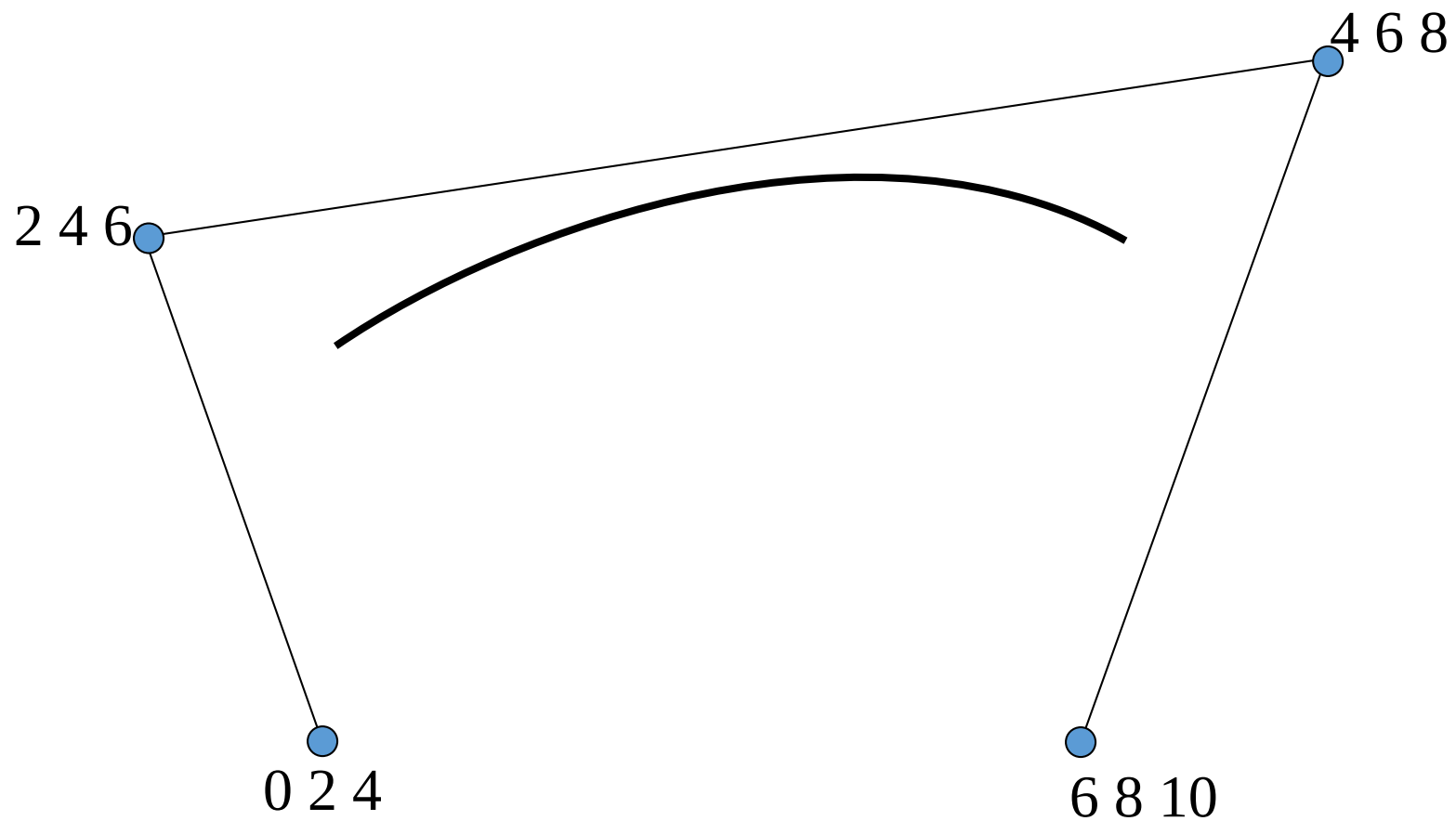
CS 418

Intro to Computer Graphics

John C. Hart

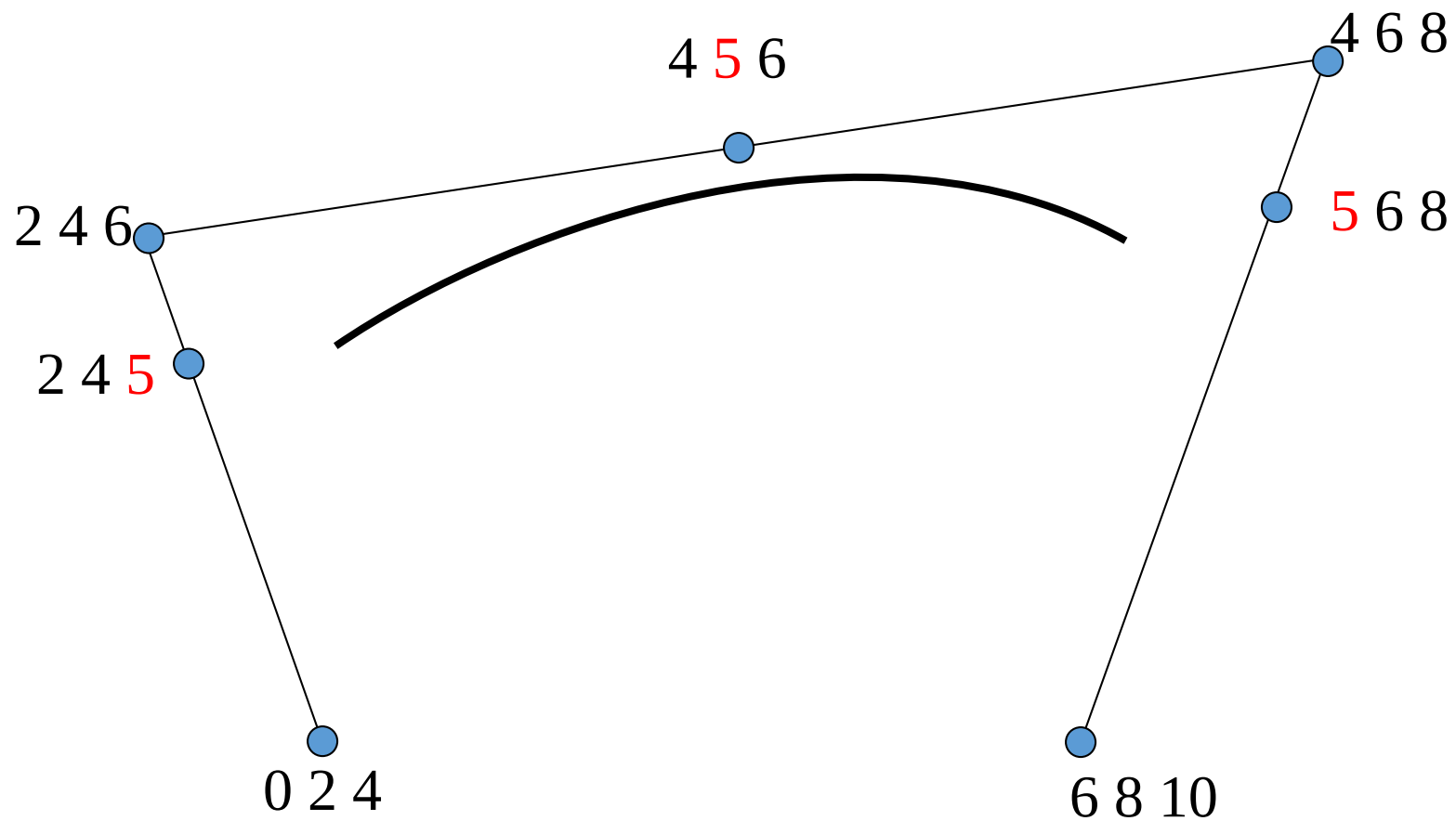
Knot Insertion

[0 2 4 6 8 10]



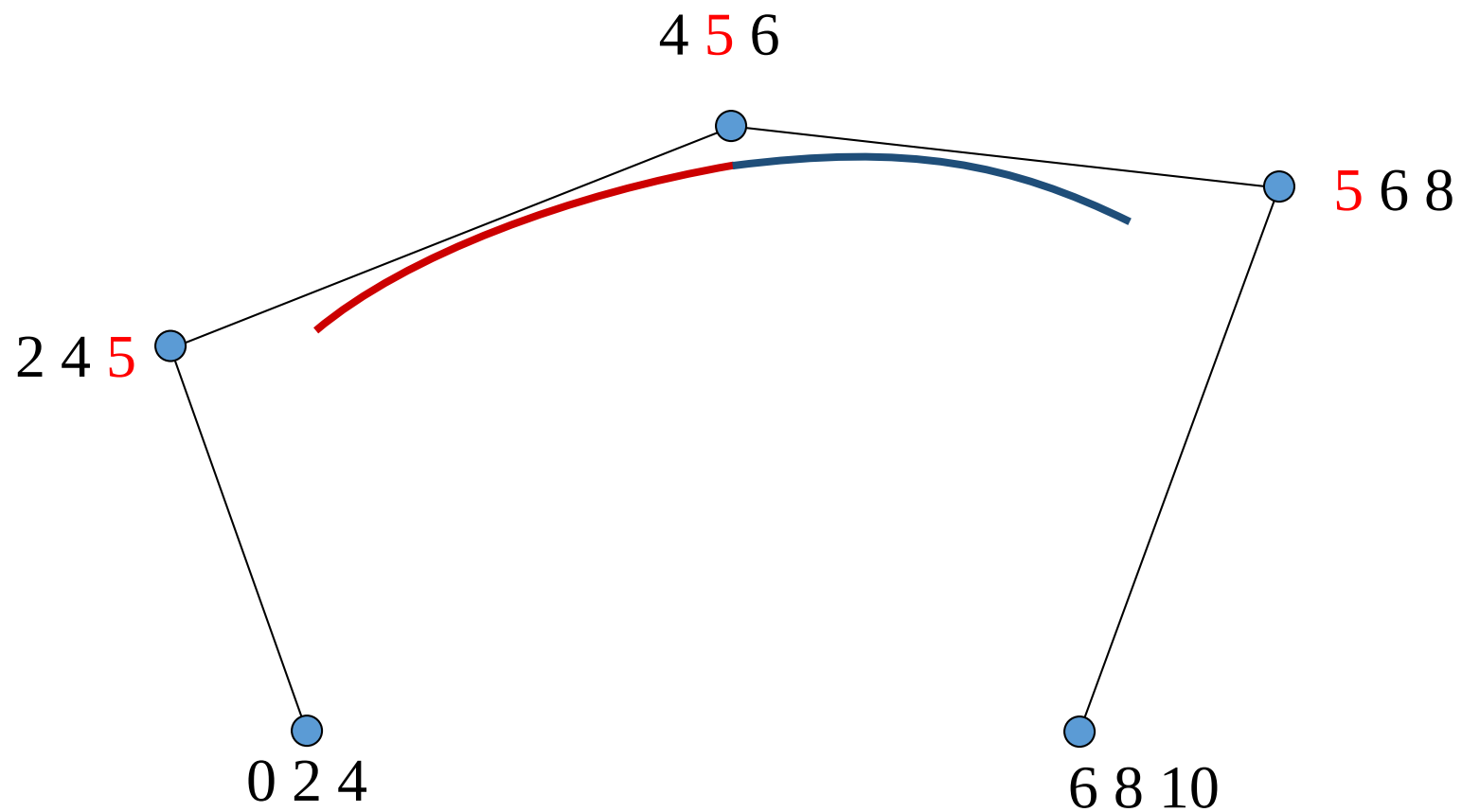
Knot Insertion

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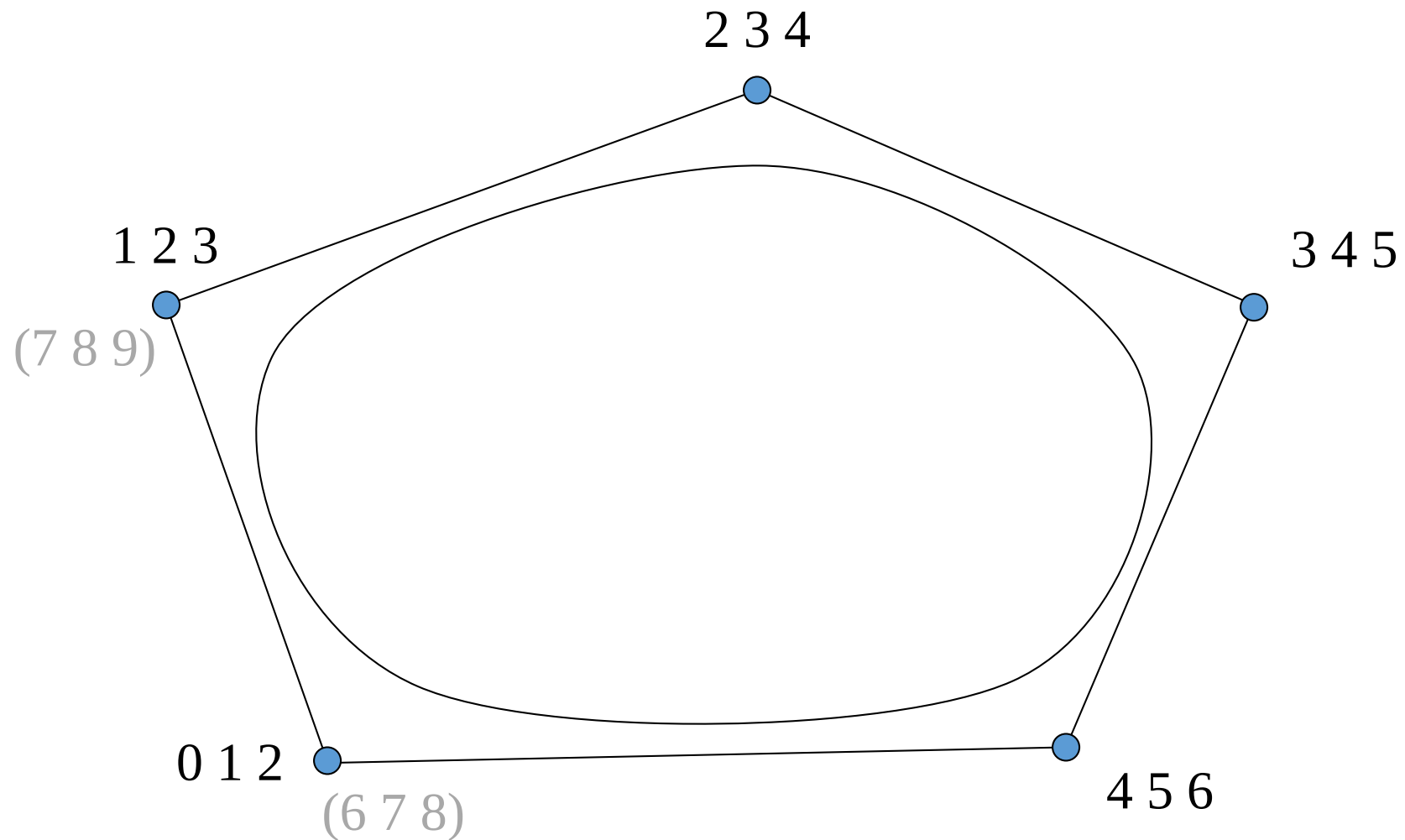


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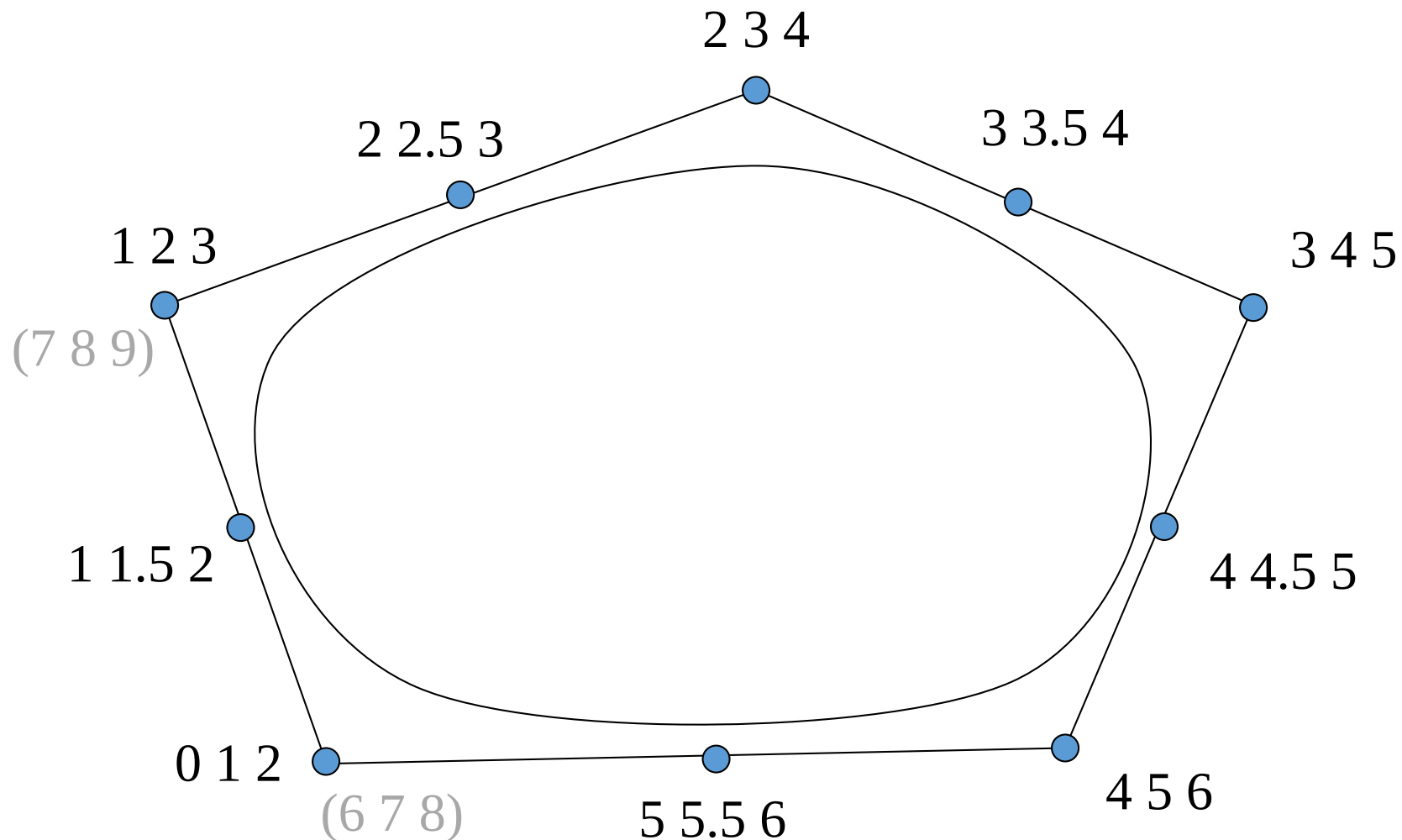
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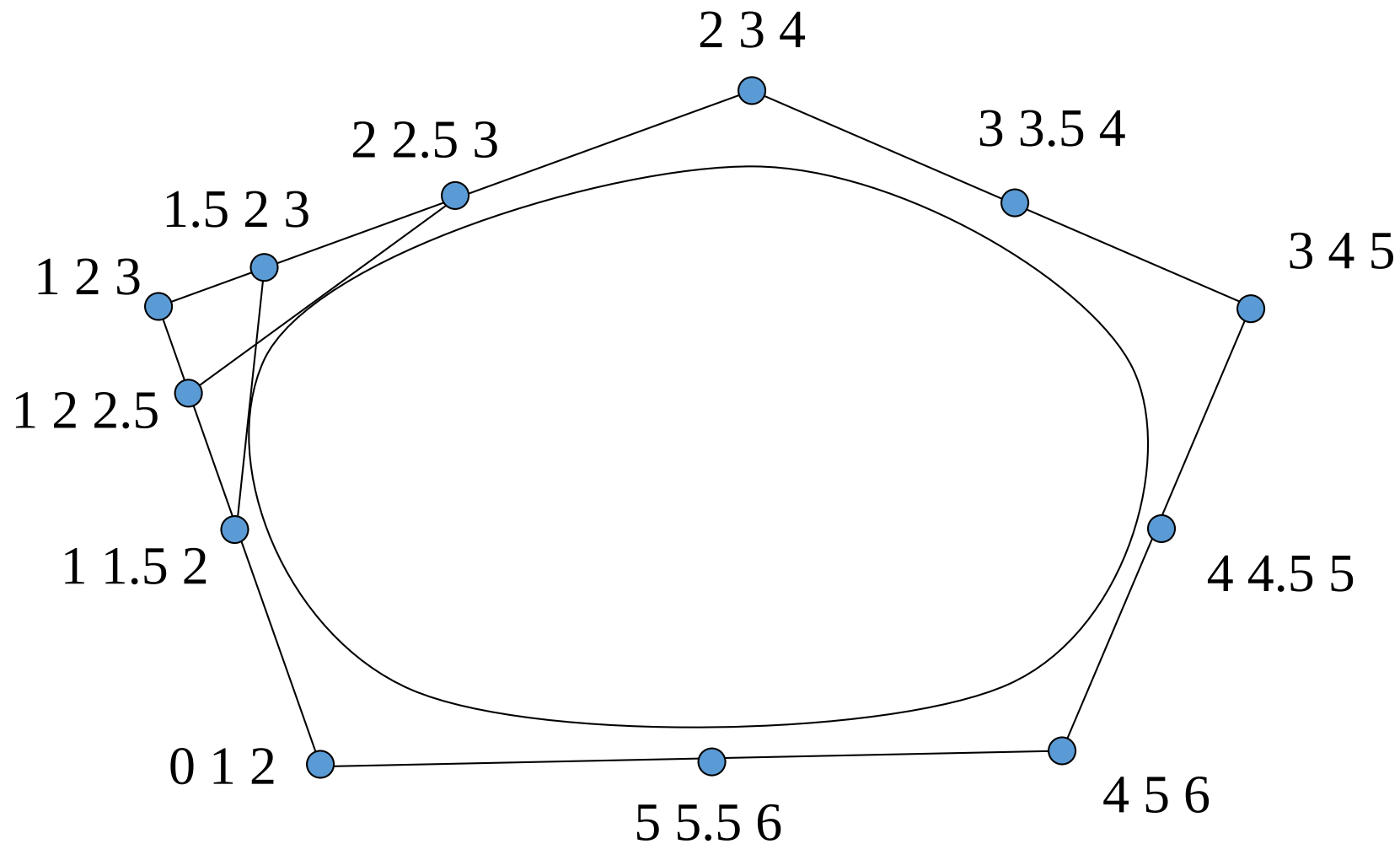
Loop Knot Insertion



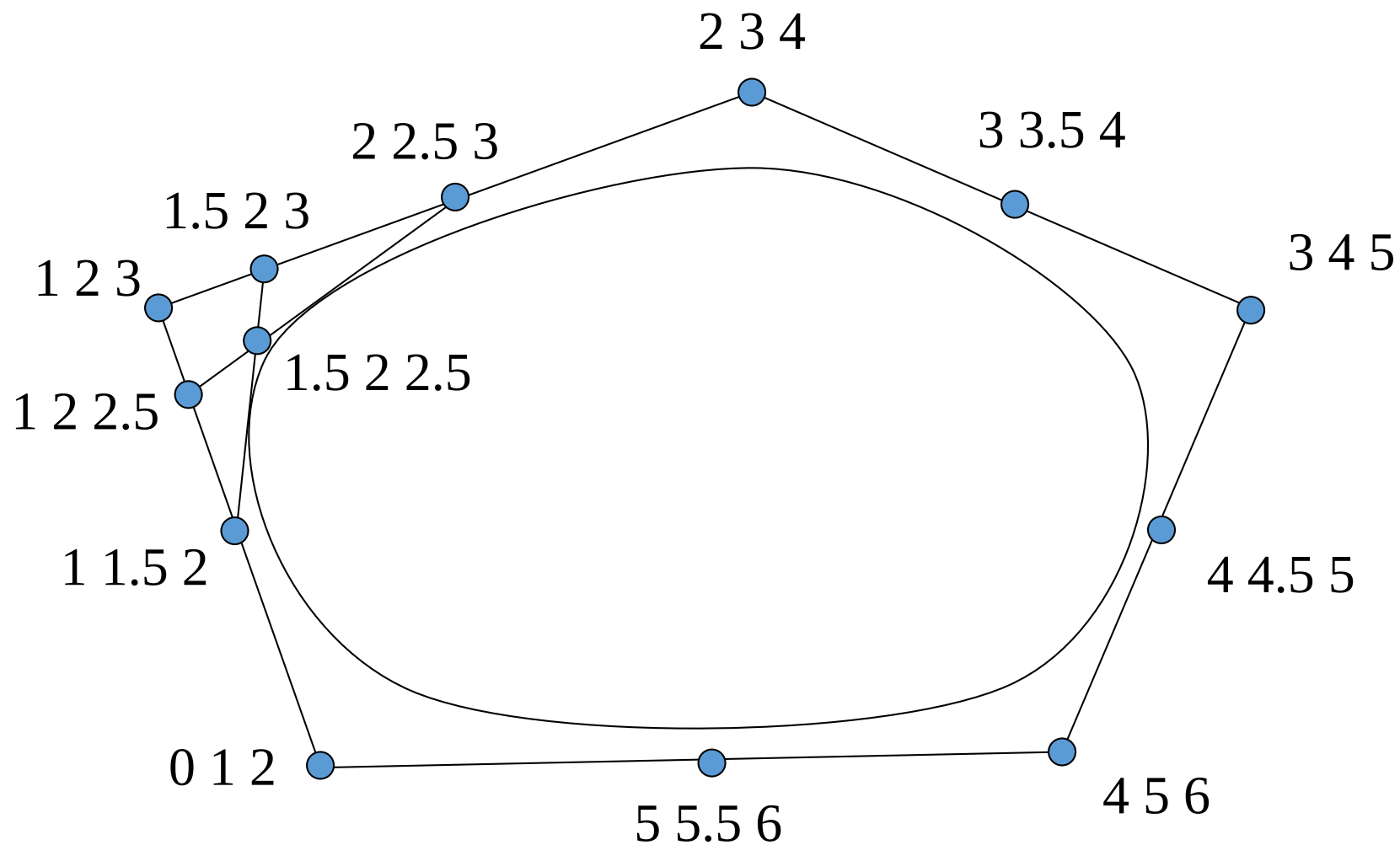
Loop Knot Insertion



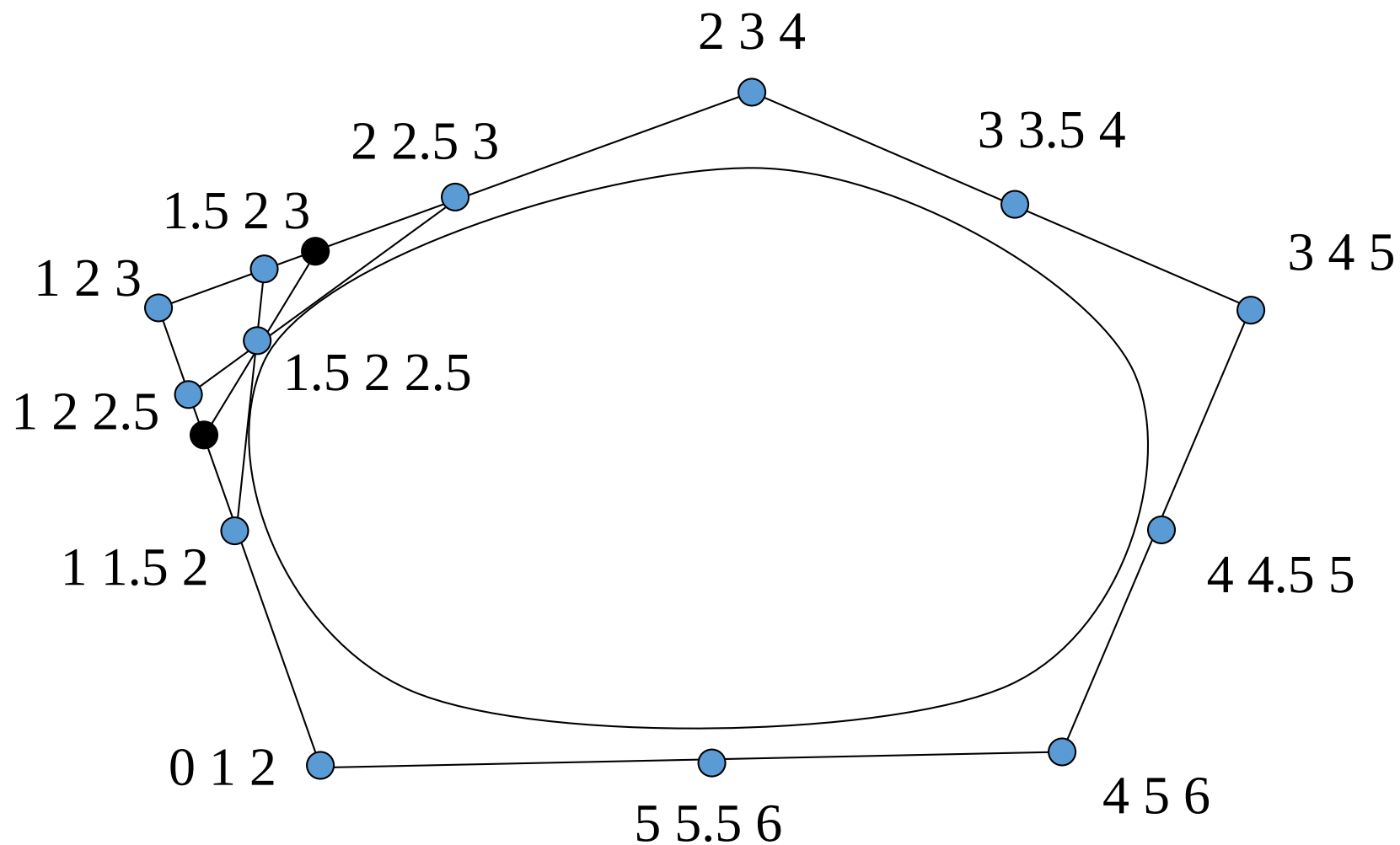
Loop Knot Insertion



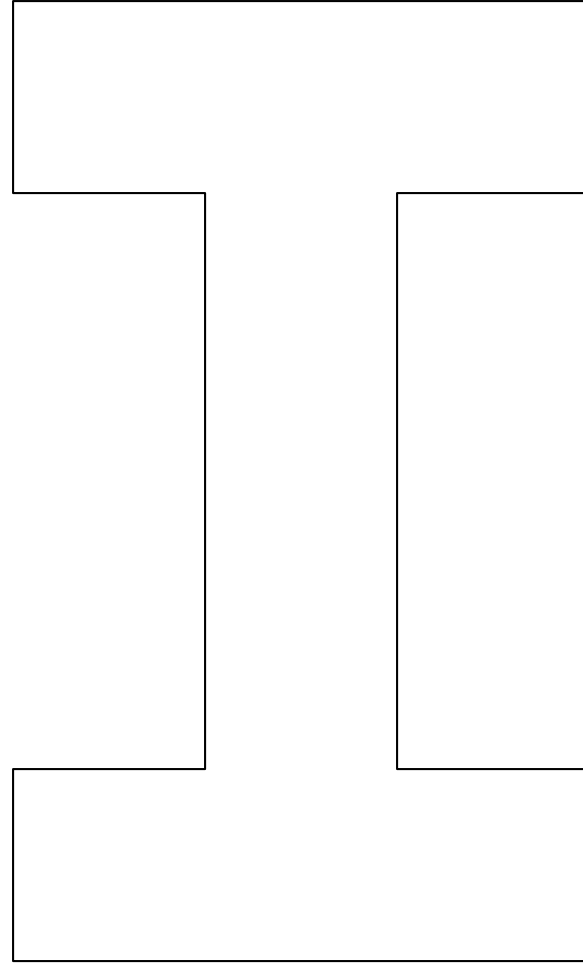
Loop Knot Insertion



Loop Knot Insertion

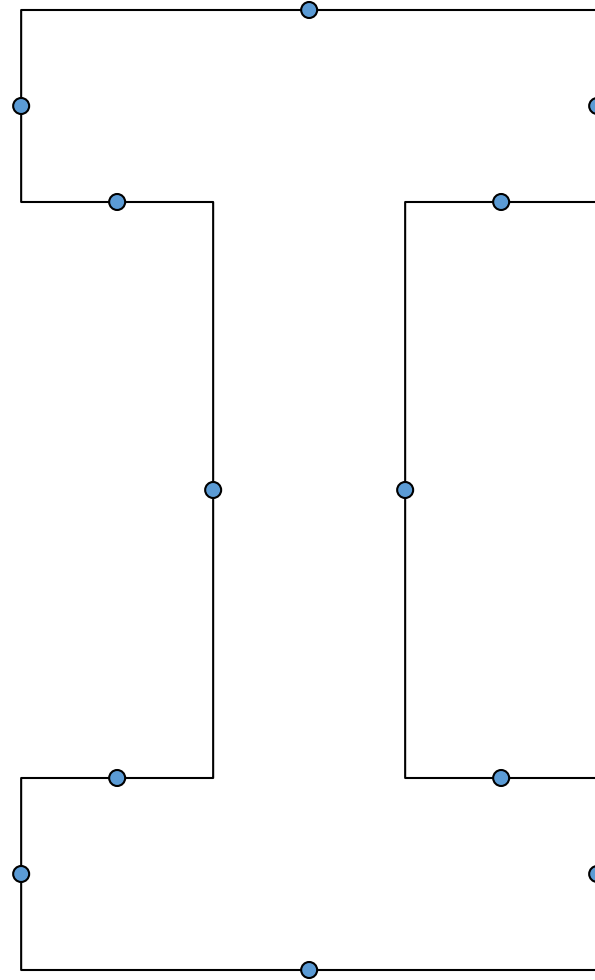


Smoothing a Polygon



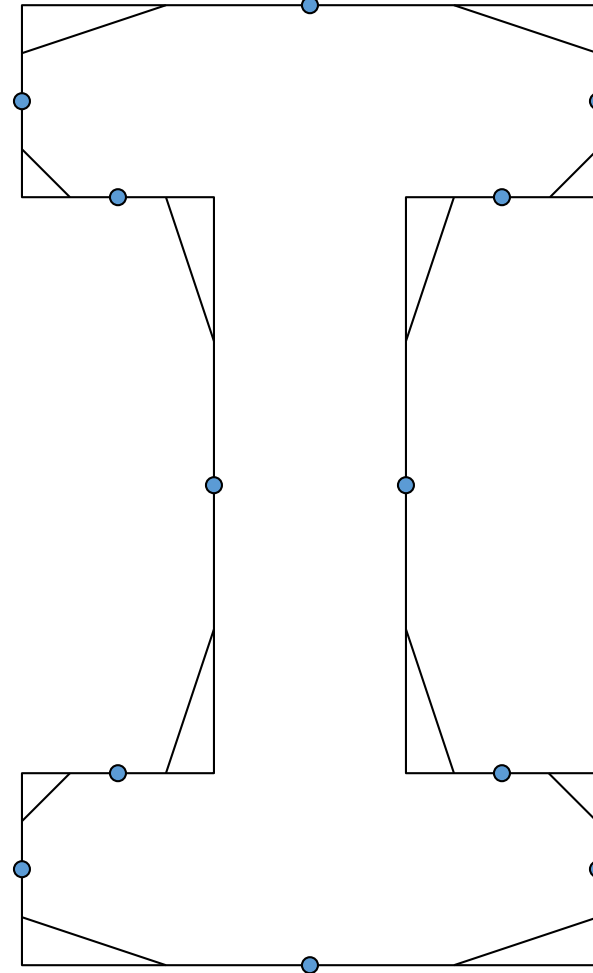
Smoothing a Polygon

1. Add edge midpoints



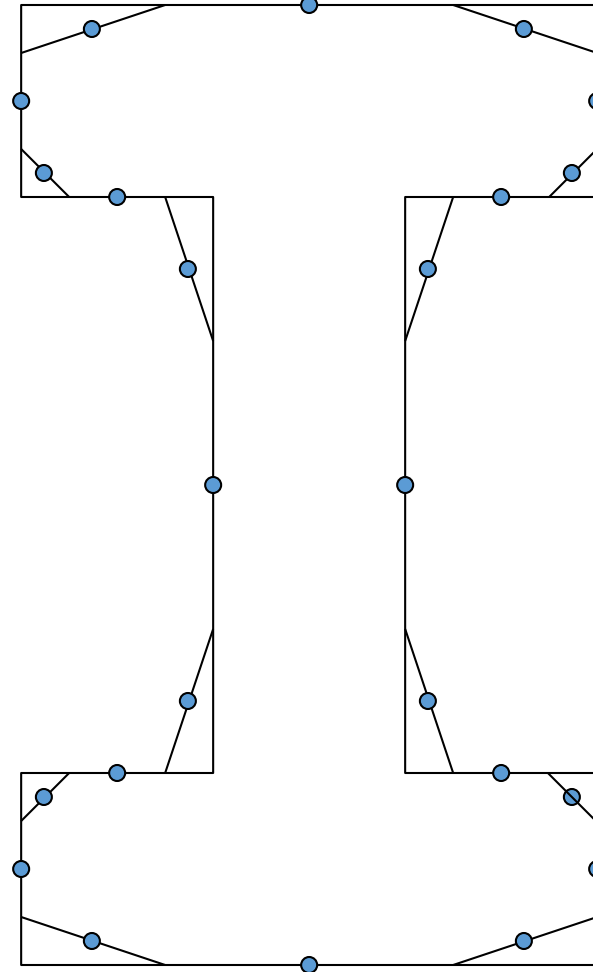
Smoothing a Polygon

1. Add edge midpoints
2. Add struts
 - Struts connect midpoints of segments from vertices to edge midpoints
 - One strut per vertex



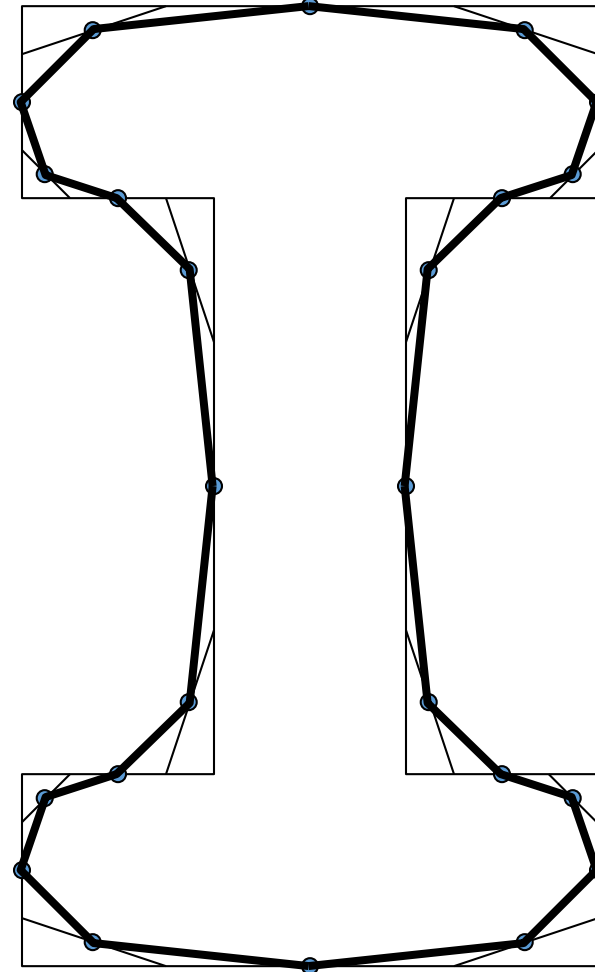
Smoothing a Polygon

1. Add edge midpoints
2. Add struts
 - Struts connect midpoints of segments from vertices to edge midpoints
 - One strut per vertex
3. Add strut midpoints



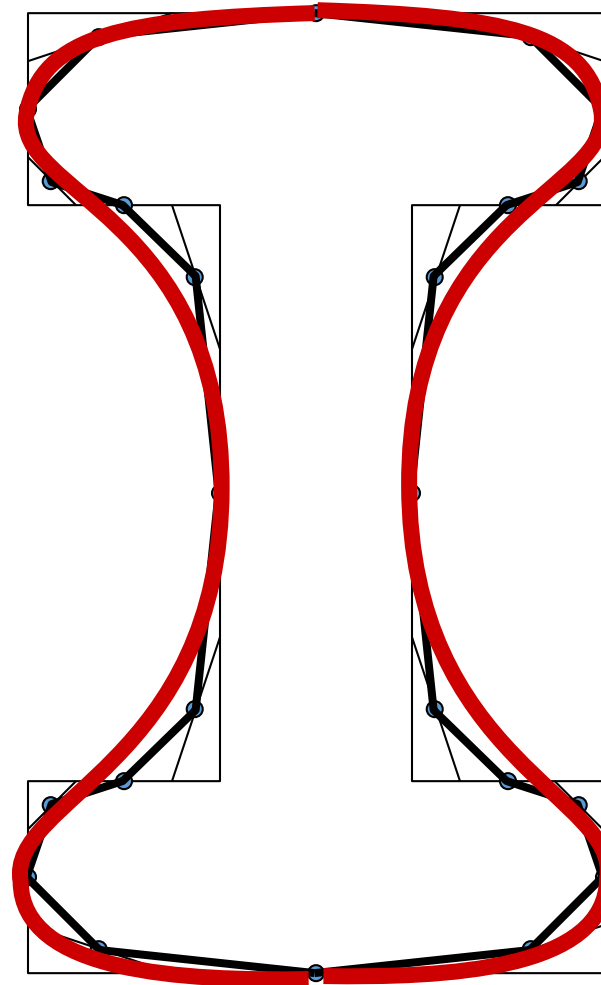
Smoothing a Polygon

1. Add edge midpoints
2. Add struts
 - Struts connect midpoints of segments from vertices to edge midpoints
 - One strut per vertex
3. Add strut midpoints
4. Connect



Smoothing a Polygon

1. Add edge midpoints
2. Add struts
 - Struts connect midpoints of segments from vertices to edge midpoints
 - One strut per vertex
3. Add strut midpoints
4. Connect
5. Repeat

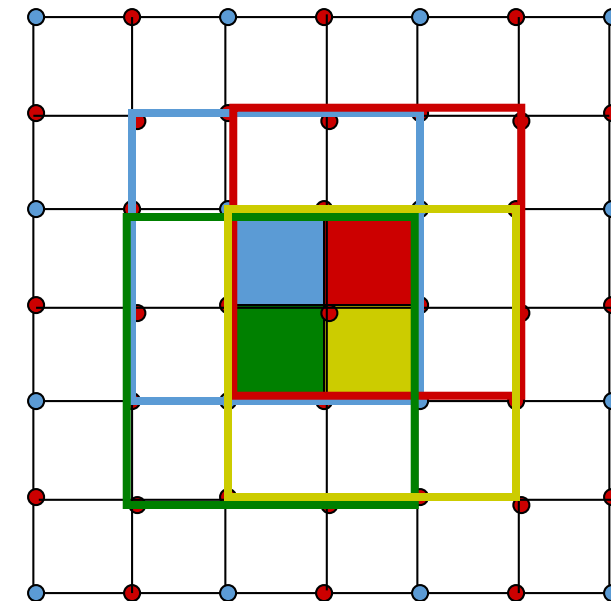
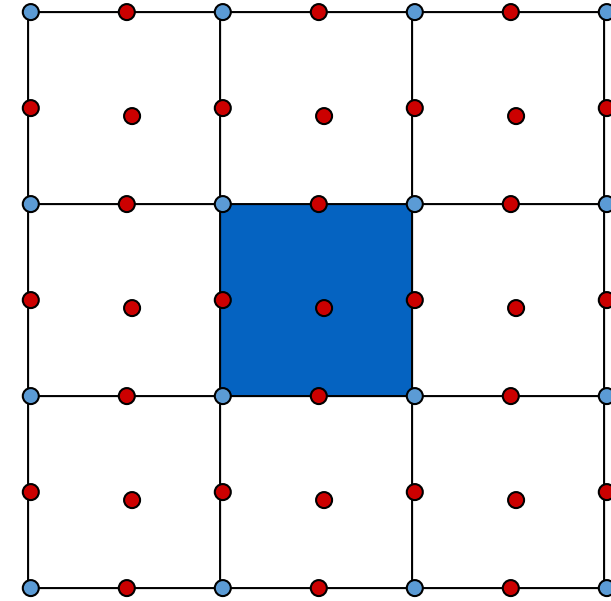


B-Spline Patches

- Tensor product of two curves

$$\mathbf{p}(s,t) = \sum_{j=0}^n \sum_{i=0}^n N_j^n(s) N_i^n(t) \mathbf{p}_{ij}$$

- Need to subdivide control points to create four sub-patches
- Need to generate new control points
 - vertex points (replacing control points)
 - edge points
 - face points

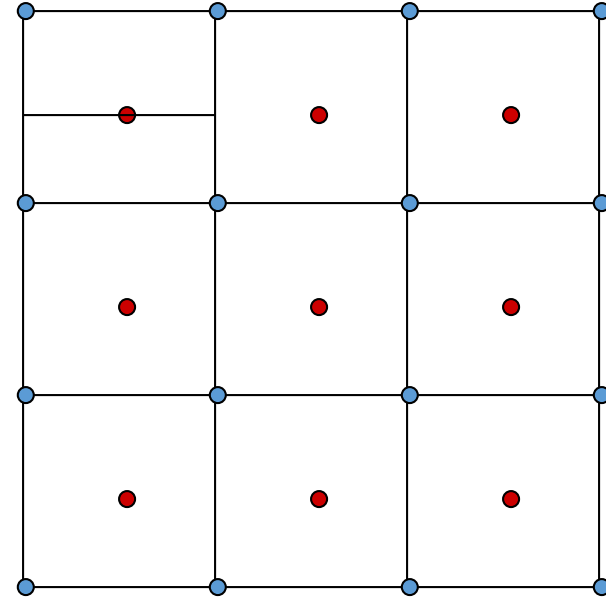


Face Points

- Approximate edge points as midpoint of control points
$$E = 1/2 \mathbf{p} + 1/2 \mathbf{p}$$
- Face point is midpoint of approximate edge points

$$F = 1/2 E + 1/2 E$$

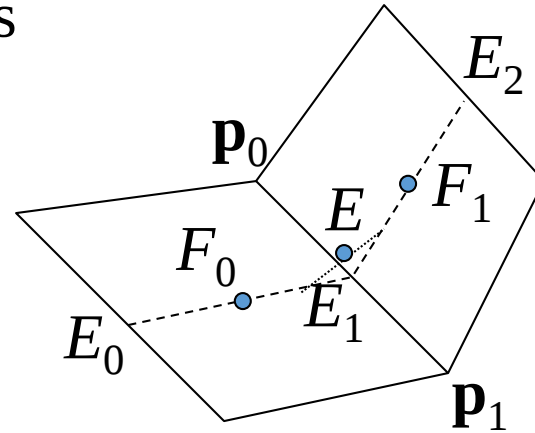
$$= 1/4 \mathbf{p} + 1/4 \mathbf{p} + 1/4 \mathbf{p} + 1/4 \mathbf{p}$$



Edge Points

- Face points are midpoints between approx. edge points
- Approx. edge point is midpoint between control points
- Actual edge point is midpoint between midpoints between approx edge point and face points

$$\begin{aligned} E &= 1/2 (1/2 (1/2 E_0 + 1/2 E_1) + 1/2 E_1) + \\ &\quad 1/2 (1/2 E_1 + 1/2 (1/2 E_1 + 1/2 E_2)) \\ &= 1/2 (1/2 F_0 + 1/2 (1/2 \mathbf{p}_0 + 1/2 \mathbf{p}_1)) + \\ &\quad 1/2 (1/2 (1/2 \mathbf{p}_0 + 1/2 \mathbf{p}_1) + 1/2 F_1) \\ &= 1/4 (F_0 + \mathbf{p}_0 + \mathbf{p}_1 + F_1) \end{aligned}$$



Vertex Points

$$V_0 = 1/4 E_0 + 1/2 \mathbf{p}_0 + 1/4 E_1$$

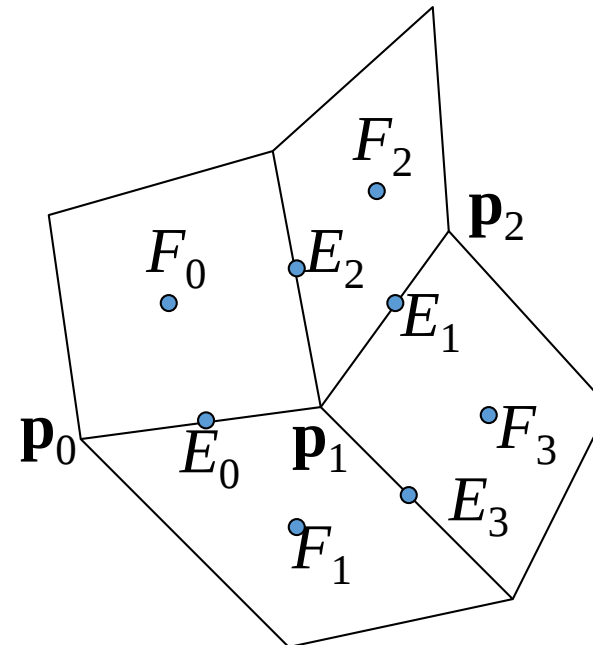
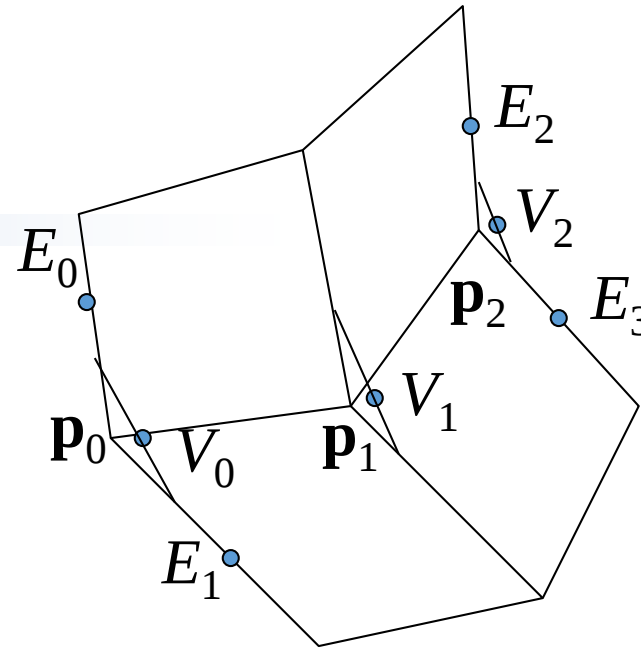
$$V_2 = 1/4 E_2 + 1/2 \mathbf{p}_2 + 1/4 E_3$$

$$V = 1/2 (1/2 (1/2 V_0 + 1/2 V_1) + 1/2 V_1) + 1/2 (1/2 V_1 + 1/2 (1/2 V_1 + 1/2 V_2))$$

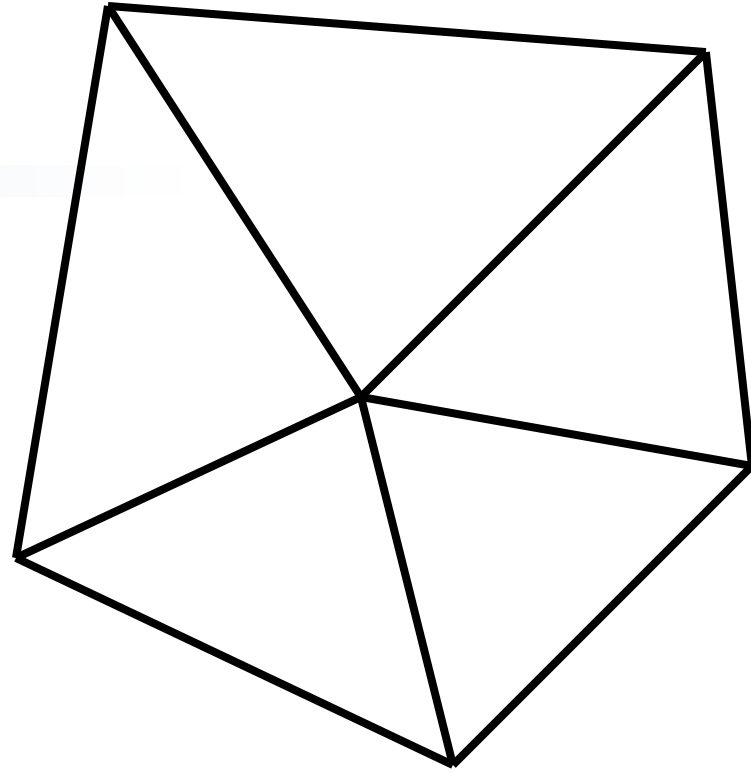
$$= 1/4 (1/4 (F_0 + F_1 + \mathbf{p}_0 + \mathbf{p}_1) + 1/4 (F_2 + F_3 + \mathbf{p}_1 + \mathbf{p}_2) + 2 V_1)$$

$$= 1/4 (1/4 (F_0 + F_1 + F_2 + F_3) + 1/4 (\mathbf{p}_0 + 2 \mathbf{p}_1 + \mathbf{p}_2) + 2/4 (E_2 + E_3 + 2 \mathbf{p}_1))$$

$$= 1/16 (F_0 + F_1 + F_2 + F_3 + 2E_0 + 2E_1 + 2E_2 + 2E_3 + 4\mathbf{p}_1)$$

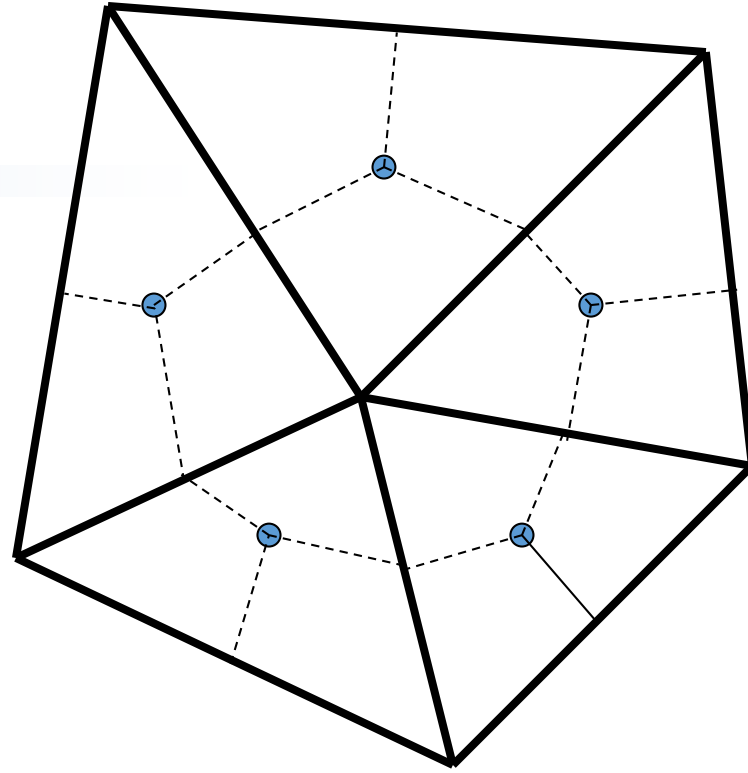


Catmull-Clark Subdiv



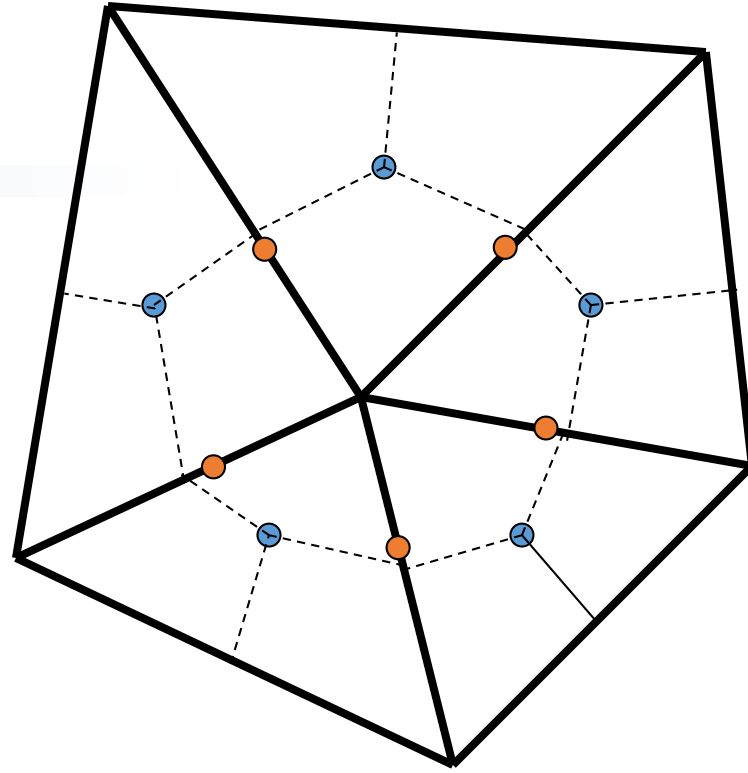
Catmull-Clark Subdiv

- Add new “face” vertex at each face centroid
centroid = average of face's vertices



Catmull-Clark Subdiv

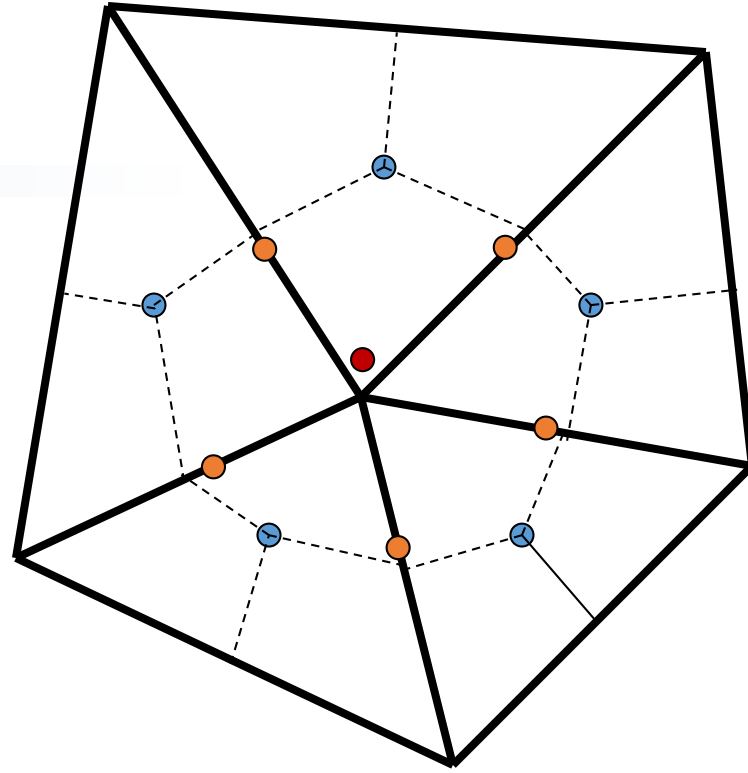
- Add new “face” vertex at each face centroid
centroid = average of face's vertices
- Add new “edge” vertex at the average of each edge's endpoints and adjacent face centroids



Catmull-Clark Subdiv

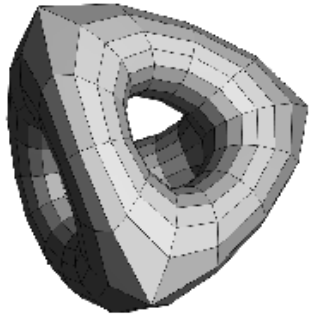
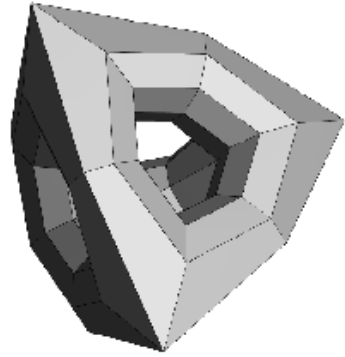
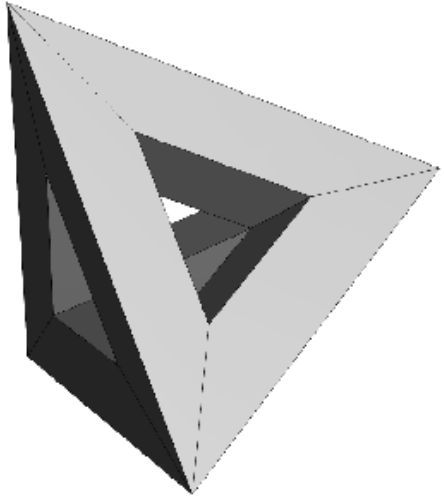
- Add new “face” vertex at each face centroid
centroid = average of face’s vertices
- Add new “edge” vertex at the average of each edge’s endpoints and adjacent face centroids
- Move each vertex to a new position that is...

$$\frac{1 \times \text{ave. adjacent face centroids} + 2 \times \text{ave. adjacent edge midpoints} + (n-3) \times \text{current vertex position}}{n}$$



where n is the valence of the vertex (# of neighboring edges, also # of adjacent faces)

Example



Creases

$f^{i+1}_j = \text{Centroid of polygon}$

$$e^{i+1}_j = (v^i + e^i_j)/2$$

- Dart vertex (one sharp edge):

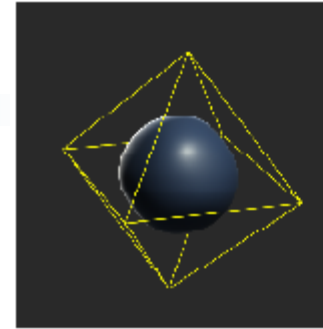
$$v^{i+1} = (n-2)/n v^i + 1/n^2 \sum_j e^i_j + 1/n^2 \sum_j f^{i+1}_j$$

- Crease vertex (two sharp edges):

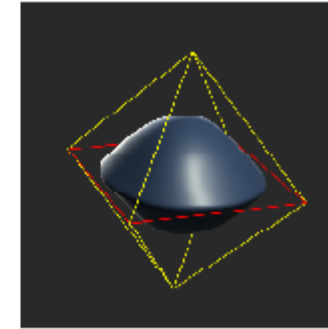
$$v^{i+1} = (e^i_j + 6v^i + e^i_k)/8$$

- Corner vertex (three or more sharp edges)

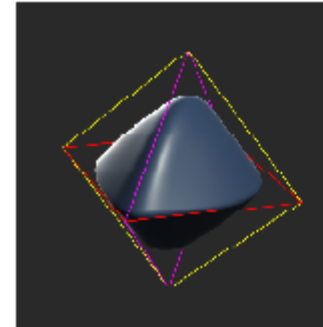
$$v^{i+1} = v^i$$



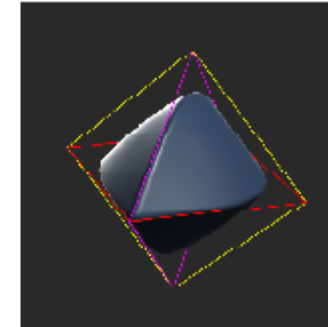
(a)



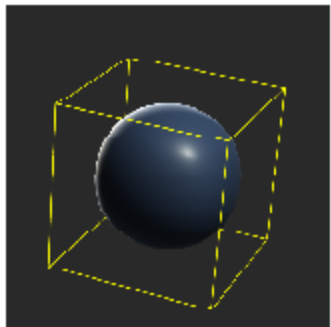
(b)



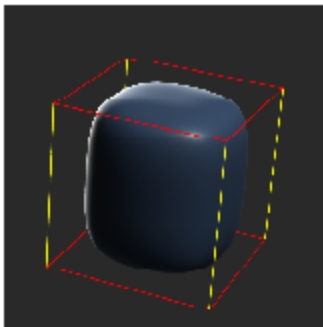
(c)



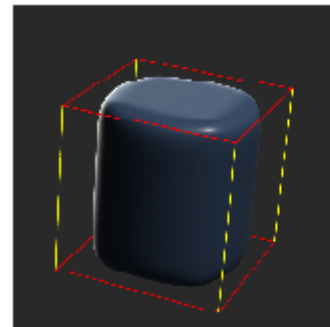
(d)



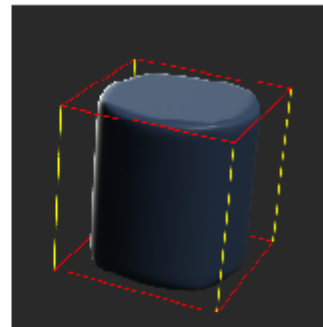
(a)



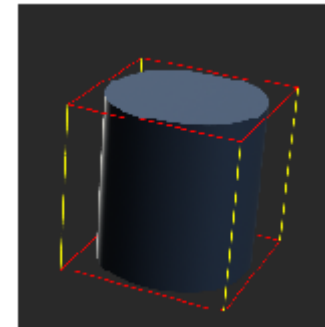
(b)



(c)



(d)



(e)

Another Example



Success?

